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## Abstract

We extend to harvesting stochastic differential equation (SDE) models in a random environment our previous work on models without harvesting concerning the resolution of the Itô–Stratonovich controversy. The resolution is obtained for the very general class of models  $dN/dt = N(r(N) - h(N) + \sigma \varepsilon(t))$ , where  $N = N(t)$  is the population size at time  $t$ ,  $r(N)$  is the (density-dependent) “mean” per capita growth rate,  $h(N)$  is the (density-dependent) harvesting effort,  $\varepsilon(t)$  is a standard white noise (representing environmental random fluctuations), and  $\sigma$  is a noise intensity parameter. Itô and Stratonovich calculus in the resolution of SDEs apparently give different qualitative and quantitative results, leading to controversy on which calculus is more appropriate and creating an obstacle on the use of this modeling approach. We show that the apparent difference between the two calculi is due to a semantic confusion based on the fallacious assumption that we are working with the same type of mean rates. After clearing the confusion, the two calculi yield exactly the same results and we obtain important common conditions for extinction and for existence of a stationary density. The resolution of the controversy is intertwined with and sheds light on the estimation issues.

**Keywords:** Harvesting models; Stochastic differential equations; Itô calculus; Stratonovich calculus; Stationary density